Dynamic statistical software: 
How are learners using it to conduct data-based investigations? 
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Abstract. Bakker (2002) identified two categories of learning software in mathematics: landscape-type software and route-type software. Route-type software was designed to guide learners through a hypothetical learning trajectory with a fairly fixed destination. Alternatively, landscape-type software is designed to support learners in conducting open-ended investigations. The use of these dynamic software tools for the learning of mathematics and statistics has gained increasing prominence in schools because of its ability to support multiple purposes defined by the user rather than the software. Little is known, however, about the diversity of approaches in which learners use these software packages to conduct investigations. This paper reports on a study of eighteen prospective secondary mathematics and science teachers’ approaches to conducting a statistical investigation using the dynamic data analysis software Fathom™ (Finzer, 2001). Three distinct approaches were identified by the research—Wonderers, Wanderers, and Answerers—each with measurable differences in their approach. This paper describes qualitative and quantitative differences in these approaches as well as their potential epistemological roots.

Introduction
Bakker (2002) identified two categories of learning software in mathematics: route-type software and landscape-type software. Route-type software was designed to support a predetermined learning trajectory with a fairly fixed destination. Alternatively, landscape-type software is identified by its ability to support learners in conducting open-ended investigations by providing them with dynamic tools—for purposes defined by the user rather than the software. The use of these dynamic software tools for the learning of mathematics and statistics has gained increasing prominence in schools because of its ability to support multiple learning routes with varied destinations. Little is known, however, about the diversity of approaches in which learners use these software packages to conduct investigations. This paper reports on a study of eighteen prospective secondary mathematics and science teachers epistemological approaches to conducting a statistical investigation using the dynamic data analysis software Fathom™ (Finzer, 2001).
Dynamic statistical learning software

Two significant shifts have been made in the teaching and learning of statistics, largely due to the influence of technology and improved theories of learning in mathematics. The first shift, beginning around the time of the first ICMI study on technology (late 1980s), was largely due the availability of computing software such as Excel, SPSS, and SAS which freed the user from tedious calculations. This change affected primarily upper secondary and tertiary instruction in formal statistics coursework (e.g., hypothesis testing, regression) as statistics was deemed too challenging for younger students. The shift provided an opportunity for instruction to evolve from a focus on learning formulas and practicing calculations towards a greater emphasis on conceptual development. These newer approaches to teaching statistics relied more heavily on technology for calculations, putting new emphasis on activity-based learning, development of conjectures and data collection, and more complex analysis of larger data sets. Like in mathematics, this opened opportunities in statistics for learners with a variety of backgrounds and learning styles, including access to powerful statistical tools for secondary students.

The drawback of employment of software to calculate statistical measures and hypothesis testing, however, was its use as a black box. That is, students would input data into the software, apply a few keystrokes to request a particular graph, calculation, or statistical test, and the results would be “spit out” by the computer. This generated much complaint by researchers that learners were developing a black-and-white view of statistics (Abelson, 1995; Gardner & Hudson, 1999), a deterministic mindset of data as a means for providing “answers” to complex questions. An opposing epistemological stance was of statistics as a tool for inquiry and analysis which acknowledges a complex world.

Statistical software packages developed in the past few years counter this “black box” approach by taking a more visual approach to statistical analysis. Software packages such as Fathom™ (Finzer, 2005) and Tinkerplots™ (Konold & Miller, 2004) were developed as learning software for doing statistics, by encouraging learners to visualize statistical relationships and develop skills in informal inference. Software packages designed to support statistical learning through open-ended investigations with data have the potential to provide users with multiple opportunities to become data wonderers, those who explore a series of “I wonder” questions as they explore data, generate and test hunches, seek insight into the phenomenon being investigated, and communicate statistical evidence for discussion and debate. Whether these software packages actually support learners in this way, however, has not been investigated. This is the goal of this paper.
Study context and method
A study conducted at a large university in the United States examined preservice teachers’ development of statistical reasoning through exploration of assessment data (Makar, 2004; Makar & Confrey, in press). The participants in the study were eighteen prospective mathematics and science teachers enrolled in an innovative course on assessment developed and taught by the authors. The course themes examined standardized and classroom testing, analysis of data using technology, and focused on developing analytic tools to highlight issues of equity in test data. Structured investigations were used throughout the course to introduce the teachers to ways in which interpreting data distributions could uncover hidden issues in high-stakes testing (Confrey & Makar, 2005).

The primary purpose of the study was to examine the interaction between the prospective teachers’ understanding of variation and distribution and their use of data as statistical evidence in an open-ended data investigation of equity in testing. This paper will focus on the epistemological approaches that the preservice teachers made use of the software Fathom™ to conduct a semi-structured data investigation. Individual interviews were videotaped as each teacher conducted an investigation in Fathom; their work on the computer was also captured. Videotape data, linked to their computer capture, were transcribed and analyzed qualitatively using Grounded Theory (Strauss & Corbin, 1998). One analysis focused on individual actions performed (e.g., observations, evaluations, and conclusions) and a second macro-level analysis documented patterns of inquiry. The action-analysis was coded by two researchers independently, with 95% agreement.

Figure 1: Representation created by nearly all participants
A random sample of data on 273 students’ scores on a state exam was provided to the prospective teachers for the investigation. The dataset contained fourteen variables (demographic information, current and previous test scores, economic level, English-language background, etc.) on sixteen-year old students of Hispanic descent from rural and urban communities. Before seeing the dataset, the teachers were asked to state a conjecture about the relative performance of Hispanic students in urban and rural schools. After stating their conjecture, they were told to investigate their conjecture in Fathom until they felt they were ready to reach a conclusion. They were given the data set but no representations (tables or graphs). Most of the participants began by creating a graph similar to Figure 1. The upper distribution is of urban student test scores and the lower distribution is of rural student test scores.

**Results**

All of the preservice teachers demonstrated facility with the software in conducting their analysis and did not find the process difficult, even though the dataset was relatively large. An analysis of the patterns of inquiry resulted in three categories of epistemological approach to conducting their investigation: Wonderers, Wanderers, and Answerers. Briefly, Wonderers are those who took an inquiry approach through cycles of “I wonder” questions: investigating the phenomenon by following a hunch, then seeking further explanations in the data through refined conjectures. Rather than using the data to investigate hunches, Wanderers, used the data to create hunches. They systematically examined graphs or summary statistics on each variable until they found a pattern that looked “interesting”. Finally, Answerers developed an initial hunch, used the data to test their hunch, and quickly ended their investigation. Further analysis is provided below for each category.

**Wonderers**

Six of the eighteen participants (33%) were classified as Wonderers. This group exhibited the kind of behaviour for which dynamic software was likely designed. They developed a conjecture of what they expected to find, then were lead through an investigation by cycles of “what if” questions, before finally settling on a conclusion (Figure 2). These questions refined their theories of understanding about the phenomenon being investigated. Their investigations in the data were guided by these theories.

![Figure 2: Model of Wonderer behaviour](image-url)
Wonderers were distinct from the other participants in two measurable ways. First, they spent a significantly longer period of time conducting their investigation \((p = 0.02)\), spending on average 26.2 minutes \((s = 12.1)\) compared to an average of 10.3 minutes \((s = 5.1)\) by the other participants. Wonderers used their time differently as well, making significantly fewer unfocused observations (statements about the data not connected to the context or conjecture) than other participants \((p = 0.03)\), making on average 2.6 observations per 5 minutes \((s = 1.2)\) compared to a mean of 4.4 \((s = 2.2)\) by other participants. Although their investigations were longer, they were focused in their exploration into the situation. Their use of the software appeared to be as a tool for inquiry – to seek insight into the context of their investigation.

**Wanderers**
The largest category of investigative behaviours was by those identified as Wanderers, with eight of the eighteen participants \((44\%)\) being categorized as such. Wanderers used the software to go through the variables available in the data, often systematically, until a relationship “popped out” at them. They did not enter their investigation with any particular theory in mind (Figure 3) nor did they conduct their inquiry in a purposeful way. They appeared to possess a belief that the data would “speak for itself” and that any relationships to be found would emerge on their own. Once a particular interesting relationship was found, they used their understanding of the context (rather than the data) to try and explain the phenomenon they had found. To them, relationships pre-existed and their job was to discover these relationships.

![Figure 3: Model of Wanderer behaviour](image)

The Wanderers used their time primarily making observations (statements not connected to the context or conjecture, such as comparing sample sizes) or drawing preliminary conclusions; few of their statements were evaluative in nature (interpreting the data with respect to the context). One of the participants summed it up well when she said during her investigation “Well, I always like to look at everything”. Their use of the technology was as a filter to “catch” potential insights. The Wanderers \((\text{mean} = 1.7, s = 0.44)\) posted a significantly higher rate of conclusion statements per five minutes \((p = 0.02)\) compared to the Wonderers \((\text{mean} = 1.0, s = 0.46)\) and
posted an average of 70% more observations per five minutes (mean = 4.4, s = 2.4) than the Wonderers (mean = 2.6, s = 1.2), although the difference was not significant (p = 0.09). In general, the Wanderers did not appear to seek insight into the phenomenon being investigated, but looked for significant results in the data devoid of context.

Answerers
The third group of behavior types recorded, the Answerers, used the software as a tool to locate a particular piece of evidence in the data to test a conjecture (theory) and then were quickly ready to draw a conclusion (Figure 4). Four of the eighteen participants (22%) were identified as Answerers. To this group, the computer was an efficiency tool that they could use to answer a question they had. This group was identified by their decision process: they looked for a particular, single piece of evidence and once they found it were satisfied that they had “answered” the question put to them. Their approach was somewhat similar to the use of computers to support statistical analysis using traditional statistical software. Rather than be a tool to explore, it was used as a way to test and decide upon the validity of a hunch.

Answerers were distinct from the other two approaches in two measurable ways. First, their investigations were significantly shorter (p < 0.01) than the other two groups, with a mean time of 5.7 minutes (s = 2.6 minutes) compared to the other participants (mean = 18.5 minutes, s = 10.8 minutes). Second, they spent significantly greater proportion of their time (p = 0.03) drawing conclusions (mean = 2.4 conclusions per 5 minutes, sd = 0.55) compared to the other participants (mean = 1.4, sd = 0.55). Their investigations were very efficient and appeared to use the software to test their conjecture rather than seek insight into the phenomenon under investigation.

Discussion
The three approaches to conducting data-based investigations with dynamic software revealed three different epistemological approaches by learners to computer supported inquiry. Wonderers approached their investigation by developing a hunch or theory about the phenomenon in question. They then used the data to both test their hunch and refine their conjecture. Wonderers possess curiosity in a way that Dewey (1910/1997) argues is vital for
reflective thinking and inquiry. “Such curiosity is the only sure guarantee of the acquisition of the primary facts upon which inference must base itself” (p. 31). It is likely that this was the model of a learner that the software developers had in mind when the software was designed, although this category emerged in only one-third of the participants. This approach reveals a learner who acknowledges that their role as an inquirer is to construct a deeper understanding of the phenomenon by creating more and more refined conjectures and demonstrates a deeper epistemological grounding.

Two different approaches, however, also emerged in the analysis. Wanderers were identified by the lack of purpose in their investigatory approach. Rather than use the data to test a hunch, they are more opportunistic in their approach, sifting through the data until something interesting caught their eye. From their findings, they try and develop a theory which they believe the data are telling them, however they explain their theory not with data but by anecdotal evidence. This epistemological approach is one in which the learner believes the data “speak for themselves” and their role as investigators is to seek out predetermined messages hidden in the data. Unlike the Wonderers who take charge of the investigation, Wanderers are led through the investigation by the data. Dewey again describes this group well, saying that their conclusions are “generated by a modicum of fact merely because the suggestions are vivid and interesting” (p. 20), and that they “find it difficult to reach any definite conclusion and wander more or less helplessly among them” (p. 36). Because the software was designed to make graph construction easy through its ‘drag-and-drop’ technology, it is possible that it may encourage this kind of behaviour in some learners. This can be of concern when the goal is to develop curious investigators, not those who wander aimlessly through graph after graph until an interesting result “pops out” at them, then explaining it post-hoc. If the number of relationships examined by the Wanderer is high, it is possible that the investigator will happen upon a significant result just by chance.

Finally, Answerers use the software as an efficiency tool by developing and testing a hunch to find an “answer” to their question. Rather than see the data as an opportunity to seek further meaning of a phenomenon, Answerers see data as having the potential to reveal answers to questions under investigation. This is consistent with Dewey’s description of learners hardened by routine: “A conclusion reached after consideration of a few alternatives may be formally correct, but it will not possess the fullness and richness of meaning of one arrived at after comparison of a greater variety of alternative suggestions” (p. 36). Dewey highlights the importance of taking one’s time in inquiry: “Time is required to digest impressions, and translate them into substantial ideas. Failure to afford time for leisure
conduce to habits of speedy, but snapshot and superficial, judgment. The depth to which a sense of the problem, of the difficulty, sinks, determines the quality of the thinking that follows” (p. 38). The Answerers’ approach is similar to one taken by many learners who use traditional software packages; they allow the user to input data, select a particular measure, graph, estimate, or model, and then present the user with the result. It requires a highly creative user with a strong conceptual understanding of both statistics and the context under investigation to use traditional software packages as something more than a black box, often bemoaned by teachers who recognize students’ black-and-white approach to statistical analysis.

The epistemological approach taken by the user is of critical importance if the goal of innovative software is to support the learner in the construction of knowledge through inquiry-based approaches. The desire is not just to guide the learner through a set of facts and procedures as it may have been previously. Rather, these software packages are developed to enable mathematical thinking that goes beyond facts and procedures. The purpose is to develop skills in inquiry as well as a mindset of mathematics as a human construction developed through the refinement of proofs and refutations (Lakatos, 1976).

The findings of this study allow us to now investigate how we can support each of these categories of user. For example, how can one encourage a Wanderer to envision a goal of structured inquiry? What kinds of tasks must be developed in order to encourage Answerers to seek further insight into their initial findings? These are important questions for the mathematics community.

The use of new dynamic software packages has provided learners with powerful tools to support a more visual approach to informal statistical inference. A critical next step is to investigate ways that learners are using these tools. This includes both acknowledging the diversity of approaches to learning that they support as well as ways in which tasks can be developed which encourage productive inquiry skills.

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References


