The examination of Computer Algebra Systems (CAS) integration into university-level mathematics teaching

Zsolt Lavicza
University of Cambridge, United Kingdom
zl221@cam.ac.uk

Although the first ICMI study was almost exclusively concerned with the integration of technology into university-level mathematics, there has been little focus on this phase of education as technology-related research has become dominated by school-level studies. Computer Algebra Systems have quietly become an integral component of university-level mathematics, but little is known about the extent of CAS use and the factors influencing its integration into university curricula. School-level studies suggest that beyond the availability of technology, teachers’ conceptions and cultural elements are key factors in technology integration into mathematics teaching and learning. In this proposal I report on an ongoing project and summarize results of the first phase of this study, which is based on interviews and observations of 22 mathematicians in three countries, Hungary, UK, and US. In addition, I outline the development of the second-phase in which a questionnaire will be sent to a sample of 3500 mathematicians in the participating countries to investigate the extent of current CAS use and to examine factors influencing CAS integration into university-level mathematics education. My research contributes to the ICMI-17 by considering cultural diversity, reflecting on actual uses of technology and addressing potential impact of CAS upon mathematics teaching and learning in universities.

The first ICMI study in 1985 reviewed the history, the potentials, the constraints, and the impact of computers on mathematics and its teaching and learning (Churchhouse et al., 1986). Despite difficulties articulated by several of its authors, the study presented an optimistic future for technology integration into mathematics education. Some years later, due to increasing accessibility to both computers and calculators, Kaput (1992) predicted that technology would become rapidly integrated into all levels of education. However, the accumulated evidence of the last fifteen years indicates that this prediction has not been realized with technology still playing a marginal role in mathematics teaching and learning (Cuban, Kirkpatrick, & Peck, 2001; Ruthven & Hennessy, 2002).

The first ICMI study was almost exclusively concerned with the integration of technology into university-level mathematics (Holton, 2001). More recently, despite a small number of studies reporting on innovative technology-assisted teaching practices and examination of university students’ learning, technology mediated mathematics education research has been dominated by school-level studies (Lagrange, Artigue, Laborde, & Trouche, 2003).

Papers in the ICMI-11 study discuss, inter alia, the role of technology in a variety of mathematics courses taught in universities, accounts of the ways...
in which technology can be used to enhance students’ learning, and the impact of technology on classroom communication (King, Hillel, & Artigue, 2001). However, the study provides neither an overview of the extent of technology use in universities nor discusses the reasons for the slowness of technology integration, preferring to offer examples of particular practices in particular universities in particular countries. The totality of the report suggests that technology use remains ‘cosmetic’ (Hillel, 2001).

Even though little is known about the state of technology use in universities, recent surveys tell us much about its use in school mathematics and the factors influencing its classroom integration at both national (Becker, 2000; Ofsted, 2004) and international levels (Gonzales et al., 2004; OECD, 2004). These studies suggest that investment in technology can enhance, but not guarantee, increased use of ICT in education, although the TIMSS 2003 study implies that funding for educational technology may not increase the actual use of ICT\(^1\) in classrooms (Gonzales et al., 2004). Other studies, which have investigated the cause of slow technology integration, suggest that, beyond the accessibility of technology and policy pressures, teachers’ beliefs and attitudes as well as cultural aspects are vital factors influencing technology integration (Hennessy, Ruthven, & Brindley, 2005; Ruthven & Hennessy, 2002). In addition, international comparative studies have reinforced the importance of cultural aspects by demonstrating that teachers’ didactical beliefs and conceptions of the subject, as well as the characteristics of their classrooms and their relation to technology, are heavily affected by teaching traditions and geographic locations (Andrews & Hatch, 2000). Results from such school-based studies may be applicable to the university setting, but a systematic investigation essential.

**Aims of the study**

Due to the paucity of university-level research outlined above, I designed a study to investigate the current use of technology together with the factors that influence its integration into university mathematics education. In my study, I focus on a specific technology application, Computer Algebra Systems (CAS), because this type of software package is the most widely used in university mathematics education. CAS is explicitly designed to carry out mathematical operations (not a general technology application such as a web-based homework system); and CAS has the potential to become a mathematical tool in students’ future studies and career (Artigue, 2005). Specifically, my study aims to examine:

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\(^1\) Information and Communication Technologies (ICT). Frequently used as a reference to technology (i.e.) in the UK.
- the extent and manner of CAS use in university mathematics departments;
- the pedagogical and mathematical conceptions of university mathematicians regarding CAS, including the factors influencing their professional use of CAS; and
- the extent to which nationally situated teaching traditions, frequently based on unarticulated assumptions, influence mathematicians’ conceptions of and motivation for using CAS.

I decided to adopt an international comparative approach in order to understand more completely different teaching traditions and subject-related conceptions (Andrews & Hatch, 2000) at the university level. The participating countries, Hungary, the United Kingdom (UK), and the United States (US), pose a variety of cultural and economic considerations. Obviously, my selection has also been influenced by my personal and professional background, as well as by my familiarity with the higher education systems of these countries. However, international comparative literature advocates the comparison of considerably dissimilar (Hungary vs. UK, US) and similar (UK, US) teaching traditions to elicit similarities and differences (Kaiser, 1999).

Methodology, methods, and preliminary results

To investigate the outlined aims I designed a two-phase study following a mixed method methodology (Johnson & Onwuegbuzie, 2004). The first, qualitative, phase of the study explored those issues that influenced university mathematics lecturers’ CAS-assisted teaching. In this phase, I interviewed 22 mathematicians, observed classes, and collected course materials in Hungary, the UK, and the US. Data were analysed by means of a grounded theory approach (Glaser & Strauss, 1967; Strauss & Corbin, 1998). Building on the results of the first phase, I am designing a large-scale quantitative study to further examine the issues that emerged from the analysis of the first phase data, to gauge the extent of CAS use in universities, and to uncover additional issues that did not surface in the initial phase of the study.

The analysis of the first phase data identified three clusters of issues:

1. Personal characteristics
2. External factors (institutional and technology issues)
3. Mathematicians’ conceptions (of mathematics, mathematics teaching/learning, CAS, CAS teaching/learning)

Many of these issues will be further investigated in the second phase of my study, but space does not permit a detailed discussion here. (A more
complete list of the subcategories of these issues can be found in Lavicza (In review). However, I highlight three of the more interesting findings of this phase below.

Firstly, similarly to results of school-level studies, academics’ conceptions, proved to be a crucial factor in technology integration into mathematics teaching. Moreover, their conceptions appear more important an influence than for schoolteachers because, due to the academic freedoms of university life, they are less prone to policy pressures. Also, mathematicians are less constrained than schoolteachers by prescribed curricula and uniform examinations. Therefore, mathematicians have better opportunities than schoolteachers to experiment with technology integration in their teaching. However, academics are frequently more concerned with research than teaching and so experiments with technology in their teaching may be seen as counterproductive.

Secondly, mathematicians’ primary use of CAS in their teaching is to enhance the transmission of mathematical concepts. Many described using CAS to illustrate mathematical concepts and I did not encounter any instance when they referred to CAS use as a motivational tool. In contrast, school-level studies report that teachers often emphasize the use of technology as motivational and classroom management tools (Hennessy et al., 2005; Ruthven & Hennessy, 2002). This result challenges the applicability of school-level findings to university settings while presenting new possibilities for collaboration that may enhance the integration of technology at all levels of mathematics education.

Thirdly, school-level studies demonstrated noteworthy differences in teachers’ conceptions of mathematics and its teaching owing to nationally situated teaching traditions (Andrews & Hatch, 2000). In my study, no distinctive teaching traditions of technology use at the university-level were identified. This may be due to the fact that the participants of my study constituted an internationally mobile group with many experienced in or aware of international university-level teaching practices and research. In addition, the use of technology in university teaching is a fairly recent endeavour. This result accords with Atweh, Clarkson, and Nebres’ (2003) idea that mathematics research and mathematics education have become an international enterprise, particularly at the university level.

For the quantitative phase of the study, I am designing a web-based questionnaire for sending to 3500\(^2\) mathematicians in Hungary, the UK, and the US. In part this will draw on the results of the first phase but will

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\(^2\) This preliminary estimate is based on the estimated population of 35,000 mathematicians in the selected three counties and the desired 20% response rate for the web questionnaire. Detailed sampling strategy is available upon request.
also attempt to uncover additional issues relating to the aims of the study. Therefore, sections of the questionnaire will enquire about:

1. the current use of CAS by selected participants;
2. participants’ personal characteristics and institutional settings;
3. participants’ variety of conceptions of CAS, CAS-assisted teaching, role of CAS in the field of mathematics and mathematics teaching.

It is my expectation that the analysis of the questionnaire data will expose relationships between participants’ personal characteristics and institutional settings, their CAS use in teaching and research, their conceptions of mathematics, and their CAS-related conceptions. Furthermore, I plan to exploit factor analytic and structural equation modelling techniques to uncover additional factors that influence CAS integration at universities.

My study will contribute to our knowledge of CAS and its use at the university level by

- providing an overview of CAS use at a large number of universities;
- identifying factors that influence CAS integration at universities and highlight similarities and differences between university- and school-level results;
- allowing insight into mathematicians’ understanding of and thinking about CAS and the impact of their teaching/cultural traditions;
- pinpointing some effects of nationally based teaching traditions of CAS use at the university level and mathematicians’ conceptions of CAS-assisted teaching.

Results of the study will enable researchers and practitioners to

- pinpoint directions for improvements and show limits of CAS applicability at universities;
- align research into local practices with international trends;
- assist in the possible development of CAS training workshops;
- improve the mathematical preparation of university students.

Once this second aspect of the study has been completed a number of possible research directions present themselves. The study may reveal issues for examination by means of a qualitative study. If, as I hope, the questionnaire proves effective, the study could be replicated in a larger set of countries. In addition, a similar study could be conducted in schools to uncover similarities and differences in university- and school-level use of technology. Furthermore, it would be possible to collaborate with mathematicians as well as school teachers to develop curricula, supporting
materials, and a variety of workshops to enhance the use of technology in mathematics education.

If invited to the ICMI-17 conference, I would be able to report on the data collection of both phases of my study and outline the preliminary results of the entire research project.

**Contributions to the ICMI-17 study**

The research reported in this proposal supports the aims of the ICMI-17 study, as set out in the discussion document, in the following ways.

My work aims to identify and analyze aspects of technology integration, primarily in universities (*diverse curricular organizations*), but also consider connections with pre-university level mathematics education. In addition, my study incorporates and investigates *cultural diversity* as it takes an international comparative approach to compare the use of technology in a less developed (Hungary) with more developed countries (UK, US) in terms of ICT resources and investment in educational technology. My study addresses well the following aim of the ICMI-17 study:

> ICMI Study 17 will also seek to take account of cultural diversity and how issues of culture alongside those related to teacher beliefs and practice all shape the way digital technologies are used and their impact on mathematics and its teaching and learning. (Hoyles & Lagrange, 2005, p.4)

My work is also in line with the following two ICMI-17 study aims:

1) to reflect on *actual* uses of technology in mathematics education, avoiding mere speculations on hypothetical prospects
2) to address the range of hardware and software with a *potential* to impact upon or contribute to mathematics teaching and learning. (Hoyles & Lagrange, 2005, p.4)

Although my study concentrates on a particular software package its results should be applicable to a wider range of applications. The examination of CAS is significant for technology-related research because such packages explicitly focus on mathematical activities and do not only reorganize communication in classrooms.

I believe that my research best fits Theme 3 – *Teachers and teaching* – as it contributes to many of the questions raised in the Study’s discussion document. Particularly, it addresses the three questions:

- How are teachers' beliefs, attitudes, mathematical and pedagogical knowledge shaped and shaped by their use of digital technologies in mathematics teaching and how are these issues influenced by access to resources and by differences in culture?
- What can we learn from teachers who use, or who have tried to use, digital technologies for mathematics teaching?
- How can teachers be supported in deciding why, when and how to implement technological resources into their teaching practices? (Hoyles & Lagrange, 2005, p.8)

References


