A model for teaching mathematics via problem-solving supported by technology

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Abstract: A model for teaching mathematics is developed based on problem-solving and the use of technology in education. The research model stems from a ten year-old intensive Calculus project containing workshops designed over Computer Algebra Systems (CAS). Emphasis is placed on distance learning attributes such as creativity, critical thinking, autonomous learning, group work and the capacity to evaluate results, study errors, and contextualize the study area. The pedagogical model is centered on the student's talents for deep learning via the solution of problems with real applications that require understanding, creativity, the use of technological tools, and the development of an appropriate language for documentation, communication and socialization. Technological tools for education include systems that allow the visualization of concepts, simulation and experimentation, operation strength and self-evaluation environments.

Teaching centered around student learning

The model studies the elements of teaching based on the student's ability to learn under the premise that the student assumes responsibility for the learning process. Accordingly the student must be responsible and motivated to learn and furthermore be willing to perform group work, utilize technology and appreciate self-evaluating environments.

A dynamic distance learning environment is generated by first considering what pedagogical activities must be developed. This is the study of the student's interaction with three learning instances, as shown in Figure 1 below.

- Working alone: The student must be able to work alone and develop autonomously the necessary cognitive abilities to learn and apply the acquired knowledge to practical tasks and open problems.
  For this aspect tools must be provided for effective self-evaluation and result verification.
Interaction with the teacher: The student must learn to rely on the teacher’s orientation and professional knowledge. For this, the teacher must guide the student in a personal fashion by evaluating the progress and providing the necessary indications, materials, and support.

Collaborative work: The student must learn to appreciate groupwork as a strategy for the construction of knowledge and problem-solving. Interaction with other students provides another way to answer arising questions while interacting in a workplace environment. For this, collaborative tools must be developed as well as verification and self-evaluating environments.

The study of the relational model leads to the choice of appropriate learning theories and pedagogical strategies to support the teaching model. These are shown around the relational model in Figure 2 and Figure 3. Finally, the model must include stages of evaluation where the instructor may evaluate the implementation of the strategies and their effect on student learning to further vary the activities accordingly.
Once the pedagogical components are established a laboratory workshop is designed. The model relies on certain talents from the interested student some of which are:

- **Discipline**: Students must be able to maintain their own learning rhythm without the physical guidance of a teacher.
• Responsibility: Students must acquire a sense of responsibility of being the principal element of the learning process. They must understand the objectives of the learning program and take them as their own.
• Scheduling: the effective management of time is essential to reach the objectives of every course.
• Curiosity, research and analysis: Students must learn to obtain and discern what is the pertinent information needed at any particular instance.

The relationship between the didactic model and the structure of the lab as a distance learning element is presented in Figure 4.

**Figure 4. Learning components of the workshop.**

**Components of the workshop:**
• Problem-solving: The solution of theoretical and applied problems is the pedagogical basic tool of learning assumed throughout the work. Problems related to the area of knowledge and to predetermined standards are chosen. Emphasis is placed on both theoretical concepts and solution methods and techniques.
• Individual work and hand calculations: Students must internalize the structure and the meaning of the problems by manipulating symbols with pencil and paper and performing the appropriate operations and calculations. Designing the strategy of the solution autonomously is very
important. Afterward, the student will be allowed to verify results with technology.

- **Group work:** Discussion is the basis of collaborative problem-solving. It allows for the presentation of different points of view and strategies of solution present in different levels of understanding. Above all it permits the creation of a language and of an abbreviated system of symbols the pertains to communication.

- **Research:** Comprises various talents such as search techniques, information confrontation and discernment of core elements, translation skills and use of technology, among others. For this probing activities are developed for the enhancement of knowledge and problem-solving.

**Technological components with Maple:**

The technological platform used was the symbolic mathematical software Maple V. Activities were presented in a standardized format. These activities will be explained further:

- **Guided exercises:** A problem is presented to the student with all phases of solution explicitly designed in Maple. The student must actively perform the operations and learn the method, algorithms, arguments and styles as well as the Maple commands. The guided problems offer guided examples of methods of proof. The student learns to do and structure mathematics while solving itemized parts of a bigger complete solution of a complex problem. Real problems are presented and their solutions are explored step by step by using Maple commands. The student must analyze the arguments and answer both conceptual and operational questions at each step. Other problems are proposed to check the apprehension of procedure. The technological platform allows the student to perform and check difficult operations.

- **Interactive exercises:** Problems presented where the student must provide the answer by reasoning, hand calculations or use of technology. These may include steps towards a solution or simply a specific answer to a problem. Feedback in provided for self-evaluation and knowledge enhancement. Feedback includes generally the complete solution to the problem sometimes with other examples. Feedback for errors may include partial solution to the problem and possibly examples and counterexamples. The interactive problems allow the student to appreciate and solidify what has been learned by offering an opportunity of self-evaluation. The tools permit the construction of solutions whose validity can be checked without the presence of the teacher. The problem is called interactive because the
student gets feedback even if the solution is wrong and hints are given towards the real solution or reasons are given for the mistakes. The problem can be solved any way and only solutions or steps of solutions are checked. This was done by programming over the software program Maple is built on. Programming allows for the prediction of errors of many types and the presentation of corresponding correct solutions to operational problems, procedure problems or conceptual problems.

- Interactive solutions to problems: Explicit versions of the previous type of exercise where solutions are framed against arguments of solution.
- Use of Internet: Mostly utilized in a bibliographical or communications context. For example, workshops can be downloaded and worked on collectively through chats or message boards.
- Feedback and self-evaluation: Classical feedback provided by an instructor is itemized and studied via explicit indicators of the actual relationship between the objectives wanted through problem-solving and the activities presented in the workshop. Some of the feedback is classical via examinations, papers or simple instructor observations. Yet the core idea is to provide the tools necessary for self-examination.

Finally, the feedback and evaluations are studied so the material can be restructured and emphasis can be placed on what the student needs to reorganize and enhance knowledge. The teacher is an important player in this part of the process and must be able to guide new understanding and abilities to the effective crystallization of basic ideas and knowledge in the area.

**The evaluation phase**

Evaluation allows for consistency and quality of the learning process and provides the teacher with the appropriate feedback on the learning of each individual student. In Figure 5 evaluation processes and components are established which will allow to design and efficient pedagogical model for self-learning.

Standards are taken as the bases of the evaluation process. These are defined as follows:

Standards are clear and public criteria that provide knowledge of what students should learn and apply in solving problems of their environment and constitute a reference point of what should be known and applied in a particular area of knowledge and a specific level of proficiency. These may depend on social, cultural and political needs of a community. Especially created in order to change the emphasis on concepts in education, mathematical and geometrical standards are included in the laboratory workshop design.
Indicators: Are characteristics or components of the standards that allow for the determination of the level of proficiency of the student in that particular task associated with the standard. The breakup of abilities allows for the evaluation of strength of knowledge and of weakness and errors. The classification of levels of abilities permits the evaluator to understand and measure the attainment of objectives in a specific fashion.

Rubric definition: A mathematical rubric is the specific talent or ability that must be attained by the student in order for learning or acquiring knowledge in an area of mathematics and a certain level of proficiency. Rubric evaluations study certain aspects or indicators and assign well-determined levels in such a way that:

- Two evaluators must reach the same results of evaluation, in this work this should include one evaluator being the student.
- The aspects are well-defined and do not allow for the consideration of irrelevant factors.
- The result must correctly and pertinently evaluate the status of the student with respect to the standards.
Evaluation: It is important to utilize both formative and summative evaluation. On the one hand, it is important to determine not only the tools that allow for the learning of a particular subject area but those that allow for the correct feedback. On the other hand, evaluation of the problem solving abilities and strategies must be included with that of the conceptual understanding and the operational proficiency in the production of a correct solution. The study of the abilities as a whole: the correct utilization of concepts learned, the abilities of creating problem solving strategies, and the abilities to assess the validity of a solution change the emphasis of a solution from that based on simple procedural and operational proficiency to one that allows the use of technology correctly in every day life. Diagnostic evaluation must be utilized for the correct planning of the activities of the course and can be applied at various stages. Well-chosen rubrics allow the ERRORS to be determined diagnostically also. Therefore every evaluation can be used as feedback for the present and future design of the course.

To conclude, the study focuses on learning via problem solving intending to use technology as a support for self-learning in providing self-evaluation tools such as: well-defined and easy to determine rubrics consisting of specific indicators of abilities tied to each problem and subject area and which are presented at an appropriate time and in a pertinent setting of a workshop; tools for communication, development of language, and research; operation, procedure and solution verification; and through these tools the personalized guidance of the teacher.

References
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