Instrumental Genesis in Dynamic Geometry Environments
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Abstract. Over the past decade, much research has been done on dynamic geometry software investigating how this virtual environment can change our perception of mathematics and doing mathematics (in particular geometry), in the hope of enriching the pedagogical practices of mathematics teaching/learning. This proposal attempts to employ Vérillon and Rabardel’s theoretical construct – instrumental genesis – to study the evolution of utilization schemes by persons engaging in dynamic geometry explorative tasks; a first step in a programme to probe deeper into how geometry is conceptualized and learnt in dynamic geometry environments (DGE). In this study, the theory of variation in the phenomenographic research approach is used as an interpretive tool. In particular, the drag-mode in dynamic geometry is perceived as an artifact, hence an instrument. Consequently, instrumentation/instrumentalization of dragging via functions of variation (contrast, separation, generalization, fusion) and dragging modalities will be a main focus of DGE instrumental genesis. In the proposal, a pair of Hong Kong pre-service mathematics student-teachers’ DGE exploration episode is presented and briefly analysed. A possible variational dragging scheme is then proposed for their process of discovery. The studying of the conversation between the two student-teachers during their collaboration in the DGE task further identified a few DGE utterances which illuminate ways to conceptualize discourses in DGE.

The proposal
Over the past decade, much research has been done on dynamic geometry software investigating how this virtual environment can change our perception of mathematics and doing mathematics (in particular geometry), in the hope of enriching the pedagogical practices of mathematics teaching/learning (see for examples, Educational Studies in Mathematics 44:1-161, 2000; International Journal of Computers for Mathematical Learning 6:229-333, 2001; Math ZDM 34(3), 2002; Leung and Lopez-Real, 2002; Lopez-Real and Leung, 2004). Dynamic Geometry Environments (DGEs) opened up a milieu for the integration of experimental mathematics into classroom didactic and consequently brought forth the role of ICT as a key contributor to mathematical discourse. A key feature of DGE is its ability to visually represent geometrical invariants amidst simultaneous variations induced by dragging activities. This dynamic tool - dragging - induces potential dialectic between the conceptual realm (abstraction) of mathematical entities and the world of virtual empirical objects. Because of
this possibility, dragging has been a major focus of research in DGE resulting in fruitful discussions on promising dragging modalities and strategies that seem to be conducive to knowledge construction (see for examples, Hölzl, 1996; Arzarello et al, 2002). Indeed, when the drag mode “acts” on a virtual empirical object in DGE, the object undergoes transformations in a domain in which the dual nature of mathematical object (Sfard, 1991) can be “lived out”. This domain is a complex network that covers various aspects of DGE and human behaviours (physical, psychological and cognitive) where artifacts/tools could be turned into instruments/psychological tools. It thus provides an integrated environment (DGE plus human) where situated (mathematical) abstraction (Noss and Hoyles, 1996) could be constructed and “behaviour of mathematics” could be studied. The transition from concrete to abstract (epistemic transformation), or vice versa (pragmatic transformation), in this integrated environment spans a zone where creativity resides. This zone is the object of study for most of the DGE research. It is where experiences and “imagination” in DGE meet, merging to shape geometrical concepts. The idea of instrumental genesis proposed by Vérillon and Rabardel (1995) is a suitable framework to begin the exploration in this transition zone.

Instrumental genesis differentiates an artifact (a man-made object/tool) from an instrument (a psychological construct) by defining the latter as formed by an artifact together with one or more associated utilization schemes that emerge from SIA (Situated Instrumented Activity). SIA (Vérillon, 2000) is an inter-activities web formed by a triad that consists of a subject (as user), an instrument (as tool) and an object (as epistemic transformation, e.g. geometrical knowledge). Tools are artifacts that can amplify or modify our abilities to transform the world around us. They are shaped and fashioned in ways that contain the potential to reify human imagination. Hence, the value of a tool is inseparable from the one who uses it, in particular, how one uses it. We learn mathematics with tools. A pair of compasses gives us a vision of the ideal circle, a ruler is a representation of straightness, a calculator enables us to see patterns behind the complexity of routine calculations, and the list goes on. A user turns a tool into an instrument for a specific mathematical task by associating with it a scheme of use. A scheme is a systematic procedure on how to use a certain tool to achieve a certain purpose. Thus, an instrument is a psychological construct in the cognitive ergonomic domain (Vérillon and Rabardel, 1995). This is the user-oriented micro-genetic process of instrumentation in instrumental genesis. At the same time, specific functionalities, even purposes, are attributed by the user to the tool (not necessarily intended by the designer of the tool) in the instrumental genesis process. Rabardel (1995) called this instrumentalization of the artifact. Hence, an instrument is a dual entity – artifactual and psychological.
A user-oriented utilization scheme is somewhat reminiscent of Kant’s *schema* – “representation of a universal procedure of imagination in providing an image of a concept” (Kant, as cited in Tasić, 2001, p.11). In a sense, a utilization scheme in instrumentation can be thought of as a local realization of a schema; however, this raises the important question of whether different utilization schemes would converge to the same concept object. Mariotti contended

“The artifact, although incorporating mathematical knowledge and integrated by appropriate utilization schemes, might not function in generating mathematical meaning; its user might not access the meaning incorporated in the artifact.” (Mariotti, 2002, p.704)

Hence a utilization scheme can be just a routine procedure for a particular task and remains a situated abstraction that might not explicitly harvest the universal mathematical meaning behind it. Instead of artifact and instrument, Mariotti favoured the Vygotskian distinction between the mediating functions of technical tools and psychological tools (or signs: a mental construct) and preferred using *semiotic mediation* as an umbrella over instrumental genesis. Instead of focusing on the evolution of a utilization scheme in instrumental genesis, meaning construction becomes the core activity in a process of internalization where technical tools are being transformed into psychological tools for the purpose of shaping new meanings. In semiotic mediation, the focus is not on distinction between artifact and instrument; rather it is on the externally or internally oriented usage of the tool that could bring about the construal of mathematical meanings.

How do we gain geometrical knowledge in DGE? This is the ultimate question that we seek to answer. Instrumental genesis and semiotic mediation are promising theoretical frameworks that could pave a path to start the journey. Any path to knowledge construction should be an experiential one. How a person experiences the world of DGE will determine the kind of knowledge that he/she gains in DGE. The utilization schemes in instrumental genesis are schemes on how to experience the potentiality of a tool/artifact. This somewhat echoes to Vergnaud’s notions of scheme and operational invariant (Vergnaud, 1999). In semiotic mediation, the process of internalization concerns the transformation of experiences; a change in the way of seeing something via a change in the nature (pragmatic, epistemic) of the tools. Experiences, discernment, variation and simultaneity are the central concepts in the phenomenographic research approach in which learning and awareness are interpreted under a theoretical framework of variation.
“The unit of phenomenographic research is a way of experiencing something, …., and the object of the research is the variation in ways of experiencing phenomena.” (Marton and Booth, 1997, p.111).

Phenomenography literally can mean the act of representing an object of study as qualitatively distinct phenomena. In particular, it concerns the second-order nondualistic (neither internal/mental nor external/physical) categories of description of the variation in ways of experiencing something (phenomena), and is about categorizing the limited number of qualitatively different ways of seeing, or experiencing, a phenomenon in a hierarchical fashion. DGE is rooted in variation in its design. It is a milieu where mathematical concepts can be given visual dynamic forms subject to our actions, powerful or not and mathematical concepts can be developed through experiencing invariance under different dimensions of variation mediated by the tool/artifact in DGE. DGE is a natural experimental ground to experience variation since it has the built-in mechanism that enables the generation (via intelligent construction and dragging by us) of various qualitatively different ways of literally seeing a geometrical phenomenon in action. In this respect, Leung (2003) and Leung and Chan (2005) made initial attempts to discuss how functions of variation (specifically contrast, separation, generalization and fusion) in the phenomenographic research approach could be realized under different DGE dragging strategies in problem-solving and conjecturing episodes. It seems to be a worthwhile research to consider the functions of variation as functions of dragging and investigate how these functions can be realized through different dragging modalities in different DGE contexts (for examples, construction, problem solving, conjecturing, proving), or vice versa. That is, how to use which dragging modalities for different functions of dragging in different DGE contexts; in other words, an instrumentation of dragging in DGE via functions of variation and dragging modalities. Hence, the overarching question is how to integrate the theory of variation in the phenomenographic research approach into instrumental genesis and semiotic mediation in the study of how geometry is learnt (experienced) in DGE, which might lead to sound pedagogical content knowledge in DGE. In particular, here are a few possible (inter-related) research questions:

1. Explore how the functions of variation interact with features (e.g. dragging modalities/strategies) in DGE and investigate how this interaction contributes to the formation of utilization schemes in the process of instrumental genesis.
2. Can ways of experiencing geometry in DGE be categorized in some sort of hierarchical schematic fashion? If yes, how does this hierarchy
contribute to the conceptualization of geometrical knowledge?

3. DGE is somewhere between physical and psychological, hence may be regarded as a kind of non-dualistic field of experience, a natural niche for the phenomenographic assumption. Does the idea of non-dualistic tool for semiotic mediation in DGE make any sense? If so, in what ways?

These are questions that need in-depth research to reach some sort of answers. Extensive research has been done on instrumental genesis in Computer Algebra System (see for examples, Artigue, 2002; Guin and Trouche, 2002, Trouche, 2004; Guin, Ruthven and Trouche, 2005); however, there seems to be a gap in the literature on instrumental genesis in DGE. In the following, we describe a portion of an episode in which two pre-service mathematics student-teachers studying at the University of Hong Kong for the Postgraduate Diploma in Education (PGDE) were taking part in a conjecture making activity in DGE. The DGE they used was C.a.R. (Compass and Ruler, a DGS developed by R. Grothmann, http://mathsrv.ku-eichstaett.de/ MGF/homes/grothmann/java/zirkel/doc_en/).

During their exploration, an implicit dragging scheme was evolved for the problem they were working on that eventually led them to the discovery of a (correct) conjecture. This dragging scheme composed of (a) key functions of variation mentioned in the discussion above and (b) dragging modalities that have been identified in research and practices (the student-teachers were not aware of these). The problem they were working on was finding a relation that was essentially the necessary condition in Ceva’s Theorem. In particular, they were given a C.a.R file (see figure on the right) in which A, B, C and P are independent draggable points while X, Y and Z are points dependent on P. The task was to find a relation connecting the lengths of the segments BX, XC, CY, YA, AZ and ZB. These two student-teachers had undergraduate degrees in mathematics-related subject areas but they didn’t know Ceva’s Theorem. They were introduced to C.a.R. in a two-hour session in the PGDE programme and they had no knowledge of the above mentioned functions of variation. At the beginning of the exploration, they spent a few minutes familiarizing themselves with the C.a.R environment and they measured the length of all the segments in question. The following is a brief outline and description of a dragging scheme that seemed to have evolved out of their exploration.
A Variational Dragging Scheme in DGE

1. *Create contrasting experiences by wandering dragging until a dimension of variation is identified.*

P was dragged to different positions inside triangle ABC in a wandering fashion while focus was put on the numerical values of the length measurements. Side BC (in particular, the point X) was chosen as a controlling variable (a dimension of variation) and while X varies as P was dragged, focus was given to the length measurements of CY, YA, AZ and ZB.

2. *Fix a value (usually a position) for the chosen dimension of variation.*

The midpoint, X’, of BC was constructed.

3. *Employ different dragging modalities/strategies to separate out critical feature(s) under the fixed value (i.e. a special case for the configuration)*

P was dragged to keep X and X’ as close as possible (see figure on the right) and attention was given to the length measurements of CY, YA, AZ and ZB. This was a guided dragging/drag-to-fit strategy. A numerical pattern was observed: the product of YA and ZB “appeared to be somewhat equal” to the product of CY and AZ (a calculator was used). Consequently, YA, ZB and CY, AZ were separated out as two related pairs. During this exploration, some DGE utterances were developed between the two student-teachers: “very difficult to control”, “try best to keep on the line”, “try to squeeze it”, etc.


Further refinement of the dragging techniques (as reflected by the DGE utterances) confirmed the speculation in 3. This is a fusion experience in which co-varying aspects (numerical values, position of P and X, the changing line segments) were simultaneously experienced together. A preliminary conjecture was proposed: when X is the midpoint of BC, the product of YA and ZB equals the product of CY and AZ.

5. *Attempt to generalize by a change to a different value for the chosen dimension of variation.*

P was dragging in a wandering fashion, however, with random patterns. For example, it was dragged horizontally for a while. After a period of fruitless exploration, the student-teachers decided to place X at a different special position: CX : CB = 1 : 3.

6. *Repeat steps 3 and 4 to find compromises or modifications (if necessary) to the conjecture proposed in step 4.*

The dragging and reasoning strategies developed in steps 3 and 4 were employed to this modified situation (a new value for the chosen dimension of
variation). The student-teachers at this moment were more experienced with the “utilization scheme” that they have developed and used it again to tackle a new situation. More DGE utterances were developed: “the point moves, all will be changed”, “it feels like the property has to do with length”, “put it on top”, etc. They discovered that the product of YA and ZB was not equal to the product of CY and AZ in this case; rather, they found that the product of YA and ZB is 3 times the product of CY and AZ. A modified conjecture was then proposed: the product of YA, CX and ZB is equal to the product of CY, XB and AZ.

7. **Generalization by varying (via different dragging modalities) other dimensions of variation**

The vertices (instead of P) A, B and C were being dragged in a wandering fashion and different values were assigned to the position of X; literally trying to see whether the modified conjecture still holds. In particular, P was dragged to move X continuously along BC. By seeing that the conjecture remained invariant under variation (by dragging) in all dimension of variation (expect when P was outside the triangle which could be seen as a design fault for the moment), the student-teachers were convinced that they had a generalized conjecture.

The 7-step variational dragging scheme described above may serve as a good starting point to investigate research question 1 and it is the intention of this proposal to undertake such an attempt. To begin with, more in-depth analysis will be done on the above episode (only the first portion of the episode was analyzed here; for the rest of the episode, the student-teachers continued the exploration in DGE and eventually proved the conjecture successfully). As mentioned above, DGE utterances were developed by the student-teachers during their exploration, it would be interesting to note the evolution of a DGE discourse as another strand of study. Further exploration tasks will be designed and persons with good mathematical background (students or teachers from secondary or tertiary) will be invited to participate in the research. It is hoped that by researching into how instrumental genesis in DGE could take place under the framework of variation, research questions 2 and 3 could become more well-defined and tangible.

**References**


34(3), 66-72.


