A Case Study Of Developing Students’ Ability 
To Design Algorithm in Logo Environment
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Abstract. The algorithmic idea has been a kind of necessary mathematics quality for modern people in this information society. In China the algorithm was represented fully as one of the new mathematics contents in the secondary level for the first time when The Standards of Mathematics Curriculum for the Senior High School was promulgated in 2003, so the research about the teaching of algorithm undoubtly has its practical implications for mathematics education. In this paper, with the conceptual framework of The Mathematics Task Framework as the research tool, an algorithmic teaching case based on LOGO software was introduced in detail, and data including observations, interviews and worksheets were collected, then the case was analyzed, and the results showed that the teaching of algorithm is feasible and effective in the LOGO environment. In the last, some beneficial implications about the instructional design of algorithm were discussed.

Introduction
An algorithm refers to the step-by-step systematic procedure used to accomplish an operation, which characterized as finiteness, definiteness, input, output, effectiveness and named by the ninth-century Arabian mathematician Mohammed al-Khowarizmi. Due to the way of mechanical operation, the algorithm hasn’t been emphasized much in the long history of mathematics education (Peng, 2004). With the rapid development of modern information technology, the algorithm begins to play a fundamental role in the development of science, technology and society, and even penetrates into every aspects of life. Being considered as a kind of necessary mathematics quality for modern people, the algorithmic idea is gradually emphasized in educational circles, which arouses the interests of the related research from mathematics education. The initial work can trace back to 1978, in which Engel outlined the comprehensive topics of the mathematics curriculum at school from an algorithmic standpoint (pp.255, 274). Similarly, Ziegenbalg argued that the concept of algorithm belongs to one of those fundamental concepts of mathematics (pp. 239, 241). At present, as the process of problem solving and mathematical application in the authentic life are highlighted in modern mathematics teaching reform, the learning and understanding of the algorithmic process is especially emphasized. It’s promoted about mathematics teaching reform in
almost every country that the traditional way of teaching algorithm should be changed to teach students to design their own algorithms and to solve realistic problems through using the algorithmic ideas, furthermore more, students should decide their own approaches and steps (Xu, 2001). Although algorithmization is a distinctive feature of mathematics in ancient China (Ma et al, 1991), the term algorithm hasn’t been represented in the mathematical textbooks for schools until 2003, when *The Standards of Mathematics Curriculum for the Senior High School* was promulgated in China and the algorithm was represented fully as one of the new contents for the first time. From it we can understand that the research about algorithm is a new issue and the reason why there is not much work in this field in China, except the research from the angles of cognitive psychology (Xu, 2003), curricular value (Li, 2004; Liu, 2003) and the significance of learning algorithm (Li, 2004). In particular, the problems about which undoubtedly should have the practical implications for mathematics education, such as how to design the mathematics teaching according to students’ real level, especially, how to integrate the information technology into the algorithmic teaching, are topics worthy of research while still are scarce.

In the following, an algorithmic teaching case in which the main classroom task is to draw a pentagram based on LOGO software will be introduced in detail, and data including observations, interviews and worksheets will also be collected, then in-depth analysis will be shown with the conceptual framework of The Mathematics Task Framework as the research tool. In the last, some beneficial implications about the instructional design of algorithm will be discussed.

**Conceptual framework**

The framework that guided much of this study stems from work done in the QUASAR project. Based partly in the ideas and research of Stein and her colleagues at QUASAR developed a framework that focuses on the cognitive demand of mathematical tasks and the various phases tasks pass through in their instructional use (Stein, Grover, & Henningsen, 1996; Stein and Smith, 1998). This framework is depicted in Figure 1.

![Figure 1: Mathematical Tasks Framework](image)
In describing the framework, Stein, Grover, and Henningsen (1996) write: “Instructional tasks are seen as passing through three phases: first, as curricular materials; second, as set up by the teacher in the classroom; and third, as implemented by students during the lesson” (p. 460). Certain cognitive demands are inherent in the way that a mathematical task is written. For example, tasks that ask students to memorize a fact or to perform an algorithm roly-poly encourage a certain type of mathematical thinking. Tasks that ask students to look for patterns, generalize, make connections, or think conceptually encourage a different kind of thinking (Stein & Smith, 1998).

In my research, the Mathematical Tasks Framework guides of my data collection, analysis, and reporting. It made sense to use this framework as a research tool because I was particularly interested in understanding how the students perform their classroom tasks pertaining to the ideas found in the Mathematical Tasks Framework. While organizing the teaching case, I was guided by my use of the Mathematical Tasks Framework in making decisions about what data to collect. These data could have been analyzed in a way that I was guided by my use of the Mathematical Tasks Framework in choosing “a coherent way of thinking about how to organize and interpret the data” (Eisenhart, 1991, p. 204).

The mathematics teaching case

Task: drawing a pentagram

The content of drawing a pentagram is chosen from the colorful LOGO-figures world, a subsection of a learning textbook, LOGO experiment, as a help of new textbooks compiled according to The Standards of Mathematics Curriculum during Compulsory Education. What should be mentioned is that although the case is taken from junior high school, the results showed that it can realize the notion of The Standards of Mathematics Curriculum for the Senior High School, so it’s helpful not only for the instructional design of algorithm in senior high school, but also for how to develop algorithmic idea in junior high school.

Teaching condition
The students have learned some basic geometry knowledge, and they can use some basic and simple LOGO orders to operate. Everyone has one computer in the well-furnished computer laboratory, which is linked by local area network, through which students and teacher can communicate freely.

Teaching process
Stage 1 Review (about 3 minutes)
The teacher guides students to review the LOGO orders, FD (FORWARD), BK (BACKWARD), RT (RIGHT), LT (LEFT), which will be used in this lesson, through the strategy of asking-answer way.

Stage 2 Learning the new LOGO order (about 10 minutes)
The students begin to learn the new order REPEAT, with the help of the teacher, through drawing the triangle, quadrangle and pentagon. In this stage, students can describe the complex recycling process and use the format:

- REPEAT 3 [FD 60 RT 120]
- REPEAT 4 [FD 60 RT 90]
- REPEAT 5 [FD 60 RT 72]

Stage 3 Exploration of the experiment (about 27 minutes)
The teacher shows a model of a pentagram and encourages students to draw it through computer.
As we all know that pentagram is a very popular geometry figure, it can be seen everywhere, such as the Chinese National Flag.
Many students are very excited when the teacher asks them to draw a pentagram, because they are familiar with it, and most of them have the experience of drawing a pentagram in a paper-pencil way. But in the face of computer based on LOGO, they feel out of place, for it’s difficult for them without considerable mathematics knowledge and the thinking way of precise expression.
The angle is the core of solving the algorithmic problem. Based on the knowledge of measure of angle in the seventh grade, the teacher guides students to review the conception of supplementary angles and adjacent angles, then tells them the characteristics of the pentagram (for example, every angle is 36°)

Students devote themselves to draw right now, and some who are good at computer can color the pentagrams.
The students find the following approaches to draw pentagrams:

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Figure 2     Regular pentagram
two algorithms
REPEAT 5 [FD 60 RT 144]
REPEAT 5 [FD 60 LT 144]
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After discussion with students about the meaning of the data 5, 60, 144, 25, 72, the teacher gives another approach of drawing.

Stage 4  Exploration and innovation (about 5 minutes)

This practice provides an approach through connecting the diagonal of the pentagon, especial and meaningful, which is a bridge of the pentagram and pentagon. It motivates students to further understand the algorithm of drawing a pentagram, and do prepare for the continuing learning of the pentagon in the future.

Discussion and conclusion
This lesson is designed from the aspect of how to explore algorithm and how to design various algorithm, aimed at developing and deepening students’ algorithmic idea through the experience of the algorithmic diversity. We’d like to analysis the obtained goal from the exploration of algorithm, algorithmic diversity and mistakes during the study of algorithm. Students’ exploration of algorithm
In this case, taking advantage of the characteristics of LOGO language, such as intuitive and easy-operate, the teacher guided students to explore the way of solving problem during the operation, to develop their own algorithmic ideas step by step and foster their ability to solve realistic
problems (drawing a pentagram from many aspects) by using the algorithmic ideas. Students can experience the algorithmic characteristics of finiteness, definiteness, input, output and effectiveness during drawing the pentagram, and deeply impressed with the input language of FD … and RT …. recycling language of REPEAT …

The following are their feeling and thoughts about exploration of the algorithm, which we got from the observation and interview in the classroom.

It’s easier to understand the approaches of drawing a pentagram through computer, when we can think carefully as well as looking at the turtle moving (the order is operated by a turtle).

It’s interesting to draw different colors and sizes of pentagrams (by using the order of color, students color red, yellow and blue for the pentagrams). Programming is not a difficult thing (students can summarize the step of drawing a pentagram as a program, and then put different numerical data into the parameter, thus making the pentagram variable in size. In fact, it has realized the transition from mathematics language to procedure language).

The turtle can finish my demands well, for which I’m highly required that no mistakes would happen during the operation.

What the above mentioned show that with the guiding of the teacher in the teaching of algorithm, it’s effective to arouse students’ enthusiasm and develop their algorithmic ideas, while letting students take active part in exploring algorithm by themselves.

**Students’ various algorithms**

There are six algorithms of drawing a pentagram developed by the students, which can be divided into two types of algorithms. According to the moving way of the turtle, the first type is based on the drawing through connecting the diagonal of the pentagram (there are two approaches, see figure 2), and the second type is based on the drawing through moving along the sides of the pentagram (there are four approaches, see figure 3). Other two students used this approach: [FD 60 RT 144 FD 60 RT 144 FD 60 RT 144 FD 60 RT 144 FD 60 RT 144 FD 60 RT 144 FD 60 RT 144], which we call regular algorithm, for it don’t need the order of REPEAT, only but FD and RT. It seems like the ordinary paper-pencil way, which can be finished step by step.

From 55 students’ procedure records, we get 49. There are 18 students who used the first or second type of algorithm, and there are 4 students who used both (see Table 1).

<table>
<thead>
<tr>
<th>Types of algorithm</th>
<th>First</th>
<th>Second</th>
<th>First &amp; Second</th>
<th>Regular</th>
<th>Wong</th>
</tr>
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</table>
Table 1: The types of the algorithms and the corresponding number of students

| Numbers of students | 18 | 18 | 4 | 2 | 7 |

According to our interview, the students who used the first or the second were affected by the order of REPEAT taught by the teacher, because it can simple the repeated process. There are only 2 students who used the regular algorithm, and we know that they didn’t catch the meaning of REPEAT, so they chose the way of step by step.

These results indicate that:
- Teacher’s action has highly active or negative effects on students’ learning of algorithm;
- The algorithmic structure of pentagram in students’ mind and LOGO language can help students to express the algorithmic model;
- Students’ past mathematics knowledge and skills have effects on the subsequent learning of algorithm. The first type of algorithm is simpler in expression than the second and more difficult to get. While the results show the number in any of these two types are the same. It is recorded by the students that because of the transition of the ordinary paper-pencil drawing, they form the habit of connecting the vertex of pentagram to draw. As for the second, it is intuitional, although relatively complex in expression and easier to find for them.

**Students’ mistakes**

There are 7 students whose algorithms are wrong, of course, they didn’t get the pentagram on computer. There are some ordinary mistakes in their algorithms, for example, procedure of REPEAT 5 [FD 25 LT 144 FD 25 RT 72] is replaced by the procedure of REPEAT 5 [FD 25 RT 144 FD 25 LT 36]. The mistakes result from the failing to understand the mathematics basic knowledge. Some students know that every angle of the pentagram is 36° with the help of the teacher, so 144° of the parameter of LT, rather than 36°, which can be learned only though computing according to the theorem of the total of internal angles of a triangle. It shows that students haven’t deeply understood the relationship between the angles. Further more, the angle of LF or RT is also important, which easy to be mistaken without a logic and precise thinking way.

From the above analysis, we can conclude that the students have obtained the expected goals and the teaching of algorithm is feasible and effective in the LOGO environment.

**Implications for the instructional design**
In this section, we’ll look back some characteristics of the instructional design of algorithm in this case.

**Roles of the teacher and students**
The learning of algorithm is a kind of uncreative learning in the psychology. In this case, the teacher didn’t indoctrinate the existing algorithm, but let students try themselves to draw a pentagram to experience the *initiative* of the algorithm through their constructive learning in the LOGO environment, thus transiting the learning of algorithm into a kind of creative learning. It’s helpful to eliminating their fear and hate to mathematics and algorithm, also promote students to understand the constructive process of algorithm (P. Dowling & R. Noss, 1990). It’s more interesting if students can create an algorithm, then it means that he or she has not only understood the algorithm, but also has known how to apply the algorithm to ordinary life.

So the role of the teacher is to create actively environment for students and to help them take part in exploring and constructing their own algorithm, rather than to teach the existing algorithm. Also, from the time of students’ participation, total to 32 minutes, we can see that students are the dominant of the learning.

**Task-directed to arouse the motivation**
The motivation can arouse students’ enthusiasm for learning, provide the direction and goal of learning. In this case, an interesting and familiar problem (drawing a pentagram) was given, catching students right now, and then resulted in requirement of recognition in the algorithm learning during the operation, thus the mathematics knowledge need to be taught and learned naturally. In the LOGO environment, the teacher can choose much more realistic problems, by using the strategies of task-directed to guide the students.

**Taking advantage of the LOGO network**
LOGO is such a good mathematical environment that it can help students toward more intuitive mathematical strategies rather than avoid analytic and it together, also it’s something of scaffold for the learning (C.Hoyles & R.Noss, 1992). Its good characteristics of friendly face, convenient language and simple operation, is easy to be learned and well liked by students. The most important is its open system that can allow students to create new orders, namely creating new algorithms. In the teaching of algorithm, we should make use of it.

On the other hand, everyone have a computer, which allows them to devote to the exploration of algorithm freely. Further more, through the monitoring system, the teacher can see everyone, and every student can ask for help
from the teacher. Students can communicate their algorithms and cooperate with one another to share resources and make progress together.

References


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¹ QUASAR (Quantitative Understanding: Amplifying Student Achievement and Reasoning) was a multi-year teacher capacity-building initiative sponsored by the Ford Foundation to change mathematics teaching practices at six middle schools ended in 1995.